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Space Station Attitude Control and Momentum Management: A Nonlinear Look

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Nonlinear design procedures are presented for obtaining attitude and angular momentum control laws for the Space Station. These are based on Lyapunov's second method for stability analysis. In the absence of disturbances, there exist four stable equilibrium points. Only one of these is desired and its stability boundary is estimated from the gravitational dynamic potential. Simulation results are presented for the current Space Station configuration for large initial deviations from the local-vertical-local-horizontal frame. The control laws obtained under the zero disturbance assumption are tested in the presence of constant disturbance torques. It is shown that the Space Station can be stabilized about a torque-equilibrium attitude and the momentum of the control moment gyroscopes is bounded, provided the disturbance is smaller than the maximum gravity gradient torque that can be produced by the Space Station.

Introduction

THE subject of attitude control of spacecraft has received considerable attention and has evolved rapidly in the past 40 years.[1-3] Linear control design methods have been used successfully to design the control laws. However, due to the presence of nonlinearities, the stability envelope of the closed-loop system can often be limited. Hence, gain scheduling is often required. This paper presents a study showing the applicability of nonlinear control design methods to the attitude control and momentum management problem. The Space Station is used as a suitable example. The Space Station is required to maintain its attitude as close to the local-vertical and local-horizontal frame (LVLH) as possible and the control moment gyroscope (CMG) momentum has to remain bounded, in the presence of aerodynamic disturbance torque.

A control design method for nonlinear systems that has gained a lot of popularity is based on Lyapunov's second method for stability analysis. The earliest work on this method has been presented by Kalman and Bertram.[4] Mortensen[5] utilized this approach for controlling rigid bodies in space. He used Rodriguez as well as Euler parameters to represent spacecraft attitude. Meyer[6] developed the idea of attitude control using Euler's theorem on rotations. More recently, Vadali and Junkins[7] used this technique for large-angle reorientation and tracking maneuvers of spacecraft with multiple reaction wheels. Wie and Barba[8] proposed the application of pulse width and pulse frequency modulation in conjunction with this approach. Variable-structure control laws have been developed by Vadali[9] and Dwyer and Sira-Ramirez.[10] Singh[11] has treated the attitude control problem of a three-rotor gyrostat in the presence of uncertainty. Large-angle maneuvering using single-gimbal CMGs has been treated by Oh and Vadali.[12] Some of the other applications of this method for controlling spacecraft attitude maneuvers can be found in Refs. 13-17.

Since the CMGs are angular momentum exchange devices, momentum has to be dumped continuously or periodically, so that momentum constraints are satisfied. Desaturating the CMGs using gravity gradient torques is the most attractive means for the Space Station, as a stable torque-equilibrium attitude (TEA) has to be maintained at all times. Hence, a continuous momentum management scheme is appropriate. Hattis[18] presents a predictive continuous momentum management concept that is based on the analytic solution to an open-loop optimal control problem. The performance index includes attitude deviation from known reference values, external torques, and CMG control torques. Woo et al.[19] present a continuous momentum management scheme and integrate the design of the attitude control law and momentum manager. Wie et al.[20] and Warren et al.[21] develop a disturbance accommodating controller that seeks TEA for the pitch and yaw axes and bounded roll attitude oscillating at orbital frequency in the presence of constant and periodic disturbances. The pitch and yaw CMG momenta are bounded and oscillate at orbital frequency, whereas the roll CMG momentum approaches zero in steady state. The presence of the oscillations is due to gyroscopic coupling between angular velocity and angular momentum, which translates into transmission zeros at orbital frequency. An optimal attitude control and momentum management scheme and its digital redesign has also been presented by Sunkel and Shieh.[22,23] Singh and Bossart[24] used the feedback linearization procedure on this problem but their pitch control law has singularities at ± 45 deg.

It is extremely important for momentum management to know that when the CMGs' momentum is zero and disturbances are absent, the station's principal axes must coincide with LVLH to achieve a TEA. In fact, under the previous assumptions, Likins and Roberson[25] have shown that there are 24 such TEAs (all of them are not stable) for a triaxial body, for which the gravity gradient torque is exactly zero. This then mandates that, in the absence of disturbances, the aim of the attitude controller should be to align the principal axes (not the body axes) with LVLH. Deviation of principal axes from LVLH will lead to slow momentum buildup, even in the absence of disturbances, and a reaction control system (RCS) input will be necessary. Therefore, even if the products of inertia are small, they cannot be neglected in the mathematical model of the station. The study of TEAs of gyrostat spacecraft with constant rotor angular momentum have been conducted by Roberson,[26] Roberson and Hooker,[27] and Longman and Roberson.[28]

This paper begins by considering the attitude control problem in a circular orbit. Euler parameters (quaternions) are utilized to describe the attitude of the station principal axes with respect to the LVLH frame. An attitude control law is derived (under zero disturbance assumption) using the fully coupled dynamic equations of motion and the Lyapunov design method. Of the four stable equilibrium points, only one is

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desired (corresponding to zero attitude error) and its stability boundary is estimated from the gravitational dynamic potential. Next, the coupled attitude control and momentum management problem is addressed for a station configuration that is gravity gradient stable. This is done only for pedagogical reasons; the current MB6 Space Station is inherently unstable. It is shown that feedback of angular rate error and CMG momentum is sufficient for this purpose. Two methods are developed to control unstable configurations. The first control law controls the attitude and pitch momentum remarkably well but needs a boundedness assumption. The second control law is not based on any such assumptions, but the closed-loop system response is slower than that obtained using the first control law. Simulation results are presented for the current Space Station configuration for large initial deviations from the LVLH frame.

To accommodate constant disturbances and achieve zero CMG angular momentum, the last step in the control design process involves representing the attitude controller using angular momentum as the control variable. The momentum manager is designed to produce the desired momentum required by the attitude controller. Simulations show that the Space Station can be stabilized about a TEA and the momentum of the CMGs is bounded, provided the disturbance is smaller than the maximum gravity gradient torque that can be produced by the Space Station. However, a proof of asymptotic stability and an estimate of the stability boundary is not provided.

Equations of Motion

Consider a rigid-body model of the station equipped with a CMG cluster. It is not particularly important to consider the exact number, type (single or double gimbal), location, and configurations of the CMGs. Singularity avoidance[12] issues relating to CMGs will also not be discussed. It is convenient to define three coordinate systems: the inertial frame $\{\boldsymbol{n}\}$, the orbital frame (also the LVLH frame) $\{\boldsymbol{o}\}$, and the Space Station principal axis frame $\{\boldsymbol{p}\}$. To achieve LVLH orientation, $\hat{\boldsymbol{p}}_1$ should point along the local horizontal (velocity vector), $\hat{\boldsymbol{p}}_2$ should point along the orbit normal, and $\hat{\boldsymbol{p}}_3$ should point along the local vertical. The orientation of the $\{\boldsymbol{o}\}$ frame with respect to the $\{\boldsymbol{n}\}$ frame can be obtained by knowing the orbital elements. The orientation of $\{\boldsymbol{p}\}$ with respect to $\{\boldsymbol{o}\}$ is represented by a set of Euler parameters, and the direction cosine matrix is given by[2,3]

$$\boldsymbol{C} = \begin{bmatrix} \beta_0^2+\beta_1^2-\beta_2^2-\beta_3^2 & 2(\beta_1\beta_2+\beta_0\beta_3) & 2(\beta_1\beta_3-\beta_0\beta_2) \\ 2(\beta_1\beta_2-\beta_0\beta_3) & \beta_0^2+\beta_1^2+\beta_2^2-\beta_3^2 & 2(\beta_2\beta_3+\beta_0\beta_1) \\ 2(\beta_1\beta_3+\beta_0\beta_2) & 2(\beta_2\beta_3-\beta_0\beta_1) & \beta_0^2-\beta_1^2-\beta_2^2+\beta_3^2 \end{bmatrix} \tag{1}$$

The Euler parameters (β_0,β_1,β_2, and β_3) differential equations are

$$\dot{\beta} = -\frac{1}{2}[\tilde{\omega}-\tilde{\omega}_f]\beta + \frac{1}{2}\beta_0(\omega-\omega_f) \tag{2}$$

$$\dot{\beta}_0 = -\frac{1}{2}(\omega-\omega_f)^T\beta \tag{3}$$

where $\beta = [\beta_1,\beta_2,\beta_3]^T$ and $\tilde{\omega}$ is the angular velocity cross-product operator defined as

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

and ω_f is the projection of the orbital rate expressed in $\{\boldsymbol{o}\}$ onto $\{\boldsymbol{p}\}$. Hence,

$$\omega_f = \boldsymbol{C}\begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix} = -n\boldsymbol{C}_2$$

where n is the circular orbit rate, and $\boldsymbol{C}_2$ is the second column of $\boldsymbol{C}$. It can also be shown that

$$\dot{\omega}_f = -\tilde{\omega}\omega_f \tag{4}$$

Hence, if ω_f and ω are equal to each other, it is clear from Eqs. (2–4) that the attitude with respect to LVLH becomes constant.

The dynamics of the station and the CMGs are represented by the following equations:

$$I\dot{\omega} = -\tilde{\omega}(I\omega+\boldsymbol{h}) + \boldsymbol{T} - \boldsymbol{u} + \boldsymbol{w} \tag{5a}$$

$$\dot{\boldsymbol{h}} = \boldsymbol{u} \tag{5b}$$

where I is the diagonal inertia matrix, and $\boldsymbol{h}$ is the CMG angular momentum vector. The gravity gradient torque vector is given by

$$\boldsymbol{T} = 3n^2\tilde{C}_3 I C_3 \tag{6}$$

where C_3 is the last column of $\boldsymbol{C}$; $\tilde{C}_3$ is the cross-product operator corresponding to C_3; $\boldsymbol{u}$ is the control vector [the actual input torque from the CMGs on the station is $-(\boldsymbol{u} + \tilde{\omega}\boldsymbol{h})$]; and $\boldsymbol{w}$ is the disturbance input, primarily due to aerodynamic effects.

Torque Equilibriums

Before attempting to control the attitude of the station, it is important to examine various possibilities by which TEAs can exist. If we assume for the present discussion that disturbances are absent, then the following possibilities might occur:

1) $\boldsymbol{h} = \boldsymbol{0}$ and $\omega = \omega_f$:

Then ω is constant and the condition for a dynamical equilibrium to exist is

$$-\tilde{\omega}_f I\omega_f + \boldsymbol{T} = \boldsymbol{0} \tag{7}$$

If Eq. (7) is satisfied, the principal axes get aligned with LVLH and the gravity gradient torque vanishes.

2) $\boldsymbol{u} \neq \boldsymbol{0}$ and $\omega = \omega_f$:

The equilibrium condition for this case is

$$\dot{\boldsymbol{h}} + \tilde{\omega}_f\boldsymbol{h} = -\tilde{\omega}_f I\omega_f + \boldsymbol{T} = \boldsymbol{0} \tag{8}$$

The gravity gradient torque in the equilibrium attitude is zero. The angular momentum component parallel to the orbit normal becomes constant, but the other two angular momenta oscillate at orbital frequency.

If the body axes are aligned with the LVLH frame, then the conditions for equilibrium can be derived easily as the following:

$$\begin{aligned} \dot{h}_1 - nh_3 &= -4n^2 I_{23} \\ \dot{h}_2 &= 3n^2 I_{13} \\ \dot{h}_3 + nh_1 &= n^2 I_{12} \end{aligned} \tag{9}$$

It can be seen that, if I_{13} is nonzero, the pitch momentum will grow linearly. The roll and yaw momenta will be oscillatory at the orbital frequency.

If constant disturbances are present, these cases can be reexamined. It is clear that LVLH orientation of the principal axes is not desirable as the orbit normal component of the angular momentum will become unbounded. On the other hand, if the angular momentum of the CMGs is zero, the equilibrium attitude must satisfy the following relationship:

$$\tilde{\omega}_f I \omega_f - \boldsymbol{T} - \boldsymbol{w} = \boldsymbol{0} \tag{10}$$

The existence of the solution to Eq. (10) in terms of the attitude variables depends on the disturbance magnitude. The maximum aerodynamic torque that can be balanced by the gravity gradient torque is determined by Eq. (10). Another important observation is that if the body axes are to be aligned with LVLH for a short while, the magnitude of the steady-state RCS torque required can be calculated easily from Eq. (10).

The attitude control problem in a circular orbit is discussed in the next section.

Attitude Control

It is essential to deal with the attitude control problem first, before addressing the combined attitude control and momentum management problem. Feedback attitude control laws for large-angle motion of spacecraft have been obtained in previous studies.[5,7,8] It has been shown that linear feedback of the Euler parameters and angular velocities can be used to stabilize a spacecraft. The works mentioned here do not include the presence of orbital motion and gravity gradient torques, nor disturbances. In this section, attitude control laws will be derived to include orbital motion and gravity gradient torques. The momentum management and constant disturbance rejection problems will be addressed later.

A form of the candidate Lyapunov function that seems natural to this system is the Hamiltonian[3] and is given by the following:

$$H = \frac{1}{2}(\omega - \omega_f)^T I(\omega - \omega_f) + \Phi \tag{11}$$

where the attitude dependent dynamic potential[3] Φ is given by

$$\Phi = \frac{3}{2}n^2 C_3^T I C_3 - \frac{1}{2}n^2 C_2^T I C_2 - \frac{1}{2}n^2(3I_3 - I_2) \tag{12}$$

It can be seen that this function is locally positive definite (around four equilibrium points) only if the moments of inertia satisfy the gravity gradient stability conditions, $\Phi > 0$. In terms of the principal moments of inertia, this is the familiar condition $I_2 > I_1 > I_3$. Controller design is relatively easy if the station configuration is stable. However, many of the evolving configurations of the station will be inherently unstable about all three axes, i.e., $\Phi \leq 0$. Hence, the candidate Lyapunov function is chosen as

$$V = \frac{1}{2}(\omega - \omega_f)^T I(\omega - \omega_f) - k\Phi \tag{13}$$

where k is a positive scalar. It can be shown that

$$\dot{\Phi} = (\omega - \omega_f)^T[-\boldsymbol{T} + \tilde{\omega}_f I \omega_f] \tag{14}$$

Further manipulations using the properties of a skew-symmetric matrix reveal that

$$(\omega - \omega_f)^T[-\tilde{\omega} I \omega - I\dot{\omega}_f] = -(\omega - \omega_f)^T[\tilde{\omega} I \omega_f]$$
$$= -(\omega - \omega_f)^T[\tilde{\omega}_f I \omega_f] \tag{15}$$

and, hence,

$$\dot{V} = (\omega - \omega_f)^T[(k+1)\{\boldsymbol{T} - \tilde{\omega} I \omega - I\dot{\omega}_f\} - \tilde{\omega}\boldsymbol{h} - \boldsymbol{u}] \tag{16}$$

It is now possible to show by utilizing Eq. (15) that the following control law renders $\dot{V}$ negative semidefinite:

$$\boldsymbol{u} = (k+1)\{\boldsymbol{T} - \tilde{\omega}_f I \omega_f\} - \tilde{\omega}\boldsymbol{h} + D(\omega - \omega_f) \tag{17}$$

where D is a positive definite matrix.

Note that $\dot{V}$ is zero only if $\omega - \omega_f$ is zero. Then the attitude is constant, Eq. (8) is satisfied, and LVLH orientation is achieved. Since a 180-deg rotation about any one of the axes also leads to the same equilibrium conditions, four possible stable equilibria can result for which the pitch axis is normal to the orbital frame. Since there exists a bounded set $\Omega_c = \{(\beta, \tilde{\omega}) \,|\, V < 3/2k\, n^2(I_3 - I_1)\}$ around each stable equilibrium point within which $\dot{V} \leq 0$ and $\dot{V} = 0$ only at the equilibrium point,

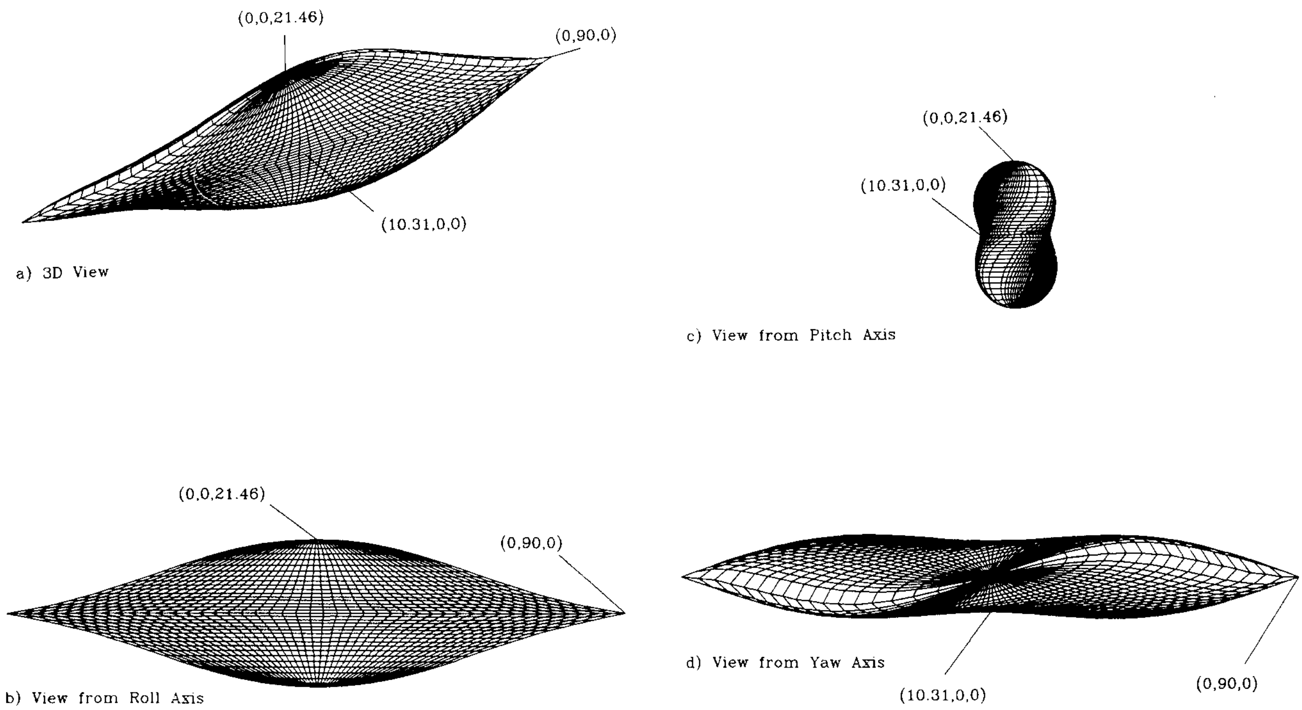

Fig. 1 Surface of maximum dynamic potential enclosing the origin.

each of them is asymptotically stable. Each Ω_c can be used to estimate the initial state (β,ω) from which every trajectory converges to the respective equilibrium point.

Since the desired equilibrium point is the origin, consider a set Ω_c around it. Figures 1 show plots of the constant dynamic potential surface for $\Phi = -3/2n^2(I_3 - I_1)$. This is the maximum value of $-\Phi$ for which the surface encloses the desired equilibrium point only. Here, the 2-3-1 (pitch-yaw-roll) Euler angle sequence has been used instead of Euler parameters. For the proposed Space Station, the principal moments of inertia I_1, I_2, and I_3 are, respectively, 19.11 E6, 2.074 E6, and 19.877 E6, all in the units of slug-ft-ft. For these values, the points of interest in Figs. 1 are ($\pm$10.31 deg,0,0), (0, $\pm$90 deg,0), and (0,0, $\pm$21.46 deg). If $\omega - \omega_f$ is zero at the initial time, then any attitude state inside the surface is guaranteed to reach the origin. The limiting value of V just obtained can be used to determine the initial attitude error that can be tolerated for a given initial value of $\omega - \omega_f$. If k is large, then larger initial angular velocity deviations can be tolerated for the same attitude deviation, but transient response becomes oscillatory.

Momentum Management and Attitude Control of a Stable Configuration (No Disturbance)

If the station configuration were stable, an elegant feedback control law can be derived to control the attitude as well as the momentum. The starting point for this design is the Lyapunov function

$$V = \frac{1}{2}(\omega - \omega_f)^T I(\omega - \omega_f) + \Phi + \frac{1}{2} \boldsymbol{h}^T k_1 \boldsymbol{h} \tag{18}$$

where k_1 is positive. The derivative of this function can be evaluated to be

$$\dot{V} = (\omega - \omega_f)^T(-\tilde{\omega}\boldsymbol{h} - \boldsymbol{u}) + \boldsymbol{h}^T k_1 \boldsymbol{u} \tag{19}$$

The equation (19) can be manipulated further by exploiting the nature of the cross-product operator, resulting in

$$\dot{V} = (\omega - \omega_f)^T(-\tilde{\omega}\boldsymbol{h} - \boldsymbol{u}) + \boldsymbol{h}^T k_1(\boldsymbol{u} + \tilde{\omega}\boldsymbol{h})$$

$$= (\omega - \omega_f - k_1\boldsymbol{h})(-\tilde{\omega}\boldsymbol{h} - \boldsymbol{u}) \tag{20}$$

It is now easy to see that the control law

$$\boldsymbol{u} = D(\omega - \omega_f) - \tilde{\omega}\boldsymbol{h} - K\boldsymbol{h} \tag{21}$$

renders the closed-loop system stable. Here, D and $K(=Dk_1)$ are positive definite matrices. Notice in Eq. (20) that, if $(\omega - \omega_f - k_1\boldsymbol{h})$ is exactly zero, then

$$\dot{\omega} = \dot{\omega}_f + k_1(-\tilde{\omega}\boldsymbol{h}) = -\tilde{\omega}(\omega_f + k_1\boldsymbol{h}) = -\tilde{\omega}\omega = \boldsymbol{0} \tag{22}$$

Thus, ω is a constant. Equation (5a) then reduces to

$$\tilde{\omega} I \omega = \boldsymbol{T} = \text{constant} \tag{23}$$

The equation (23) is satisfied only at the equilibrium points where $\omega = \omega_f$ and this implies that $\boldsymbol{h} = \boldsymbol{0}$.

Notice that, if the station is gravity stable, the control law is independent of system parameters. This control law will provide attitude control as well as momentum management if external disturbances are absent. Feedback of the integral of CMG momentum has to be introduced into the control law to bring the momentum value to zero in the presence of constant disturbances. Attitude control and momentum management of an unstable configuration is treated next. The disturbance accommodation problem will be addressed later.

Attitude Control and Momentum Management of an Unstable Configuration (No Disturbance)

Control Law 1-A

Consider a modified version of the Lyapunov function given in Eq. (13):

$$V = -k\Phi + \frac{1}{2}(\omega - \omega_f)^T I(\omega - \omega_f) + \frac{1}{2} z^T K_1 z \tag{24}$$

where

$$z = (k+1)I(\omega - \omega_f) + k\int \tilde{\omega}\boldsymbol{h}\,\mathrm{d}t + k\boldsymbol{h} \tag{25}$$

and K_1 is a positive definite matrix. It can be verified that

$$\dot{z} = (k+1)\{\boldsymbol{T} - \tilde{\omega} I\omega - I\dot{\omega}_f\} - \tilde{\omega}\boldsymbol{h} - \boldsymbol{u} \tag{26}$$

The motivation behind this definition of the internal state variable z will become clear from Eq. (16). Notice that, since $\dot{z}$ is already a factor in the derivative of the Lyapunov function, adding $\frac{1}{2} z^T K_1 z$ to V will naturally produce in $\dot{V}$ another term with $\dot{z}$ as a factor. This is exactly what is needed to include z in the control law that introduces momentum feedback. The derivative of the Lyapunov function is

$$\dot{V} = (\omega - \omega_f + K_1 z)^T \dot{z} \tag{27}$$

Following the usual procedure, the control law can be obtained as

$$\boldsymbol{u} = D(\omega - \omega_f) + Kz + (k+1)\{\boldsymbol{T} - \tilde{\omega} I\omega - I\dot{\omega}_f\} - \tilde{\omega}\boldsymbol{h} \tag{28}$$

where D is a positive definite matrix, and $K = DK_1$. $\dot{V}$ is zero in an interval if $\omega - \omega_f + K_1 z$ is zero. During this interval, it can be shown that the following statements are true:

$$\omega - \omega_f = \text{constant}$$

$$\Phi = \text{constant}$$

$$\boldsymbol{T} - \tilde{\omega} I\omega - I\dot{\omega}_f = \boldsymbol{0}$$

$$\boldsymbol{u} + \tilde{\omega}\boldsymbol{h} = \boldsymbol{0}$$

If $\omega - \omega_f$ is constant but nonzero, then $C^T(\omega - \omega_f)$ is also constant. The only possible motion is about the pitch axis. If this is so, then Φ cannot be constant. Thus, in order to satisfy the previous conditions, ω must be equal to ω_f and Eq. (8) is satisfied. Thus, LVLH orientation is achieved. A sufficient condition for the stability of the desired equilibrium point is given by $V(t_0) < 3/2\, kn^2(I_3 - I_1)$. Pitch CMG momentum will tend to zero but roll and yaw momenta will be oscillatory due to the fact that $\boldsymbol{u} + \tilde{\omega}_f\boldsymbol{h}$ is zero in steady state.

The subsequent design procedure is based on system decomposition into two connected subsystems: the attitude controller and the momentum manager. This method is motivated by the principles of variable-structure control.[9,10,28] The angular momentum required by the attitude controller [Eq. (28)] is provided by the momentum manager. Equation (28) can be integrated and written as

$$\boldsymbol{h} = -G_1\int(\omega - \omega_f)\,\mathrm{d}t - G_2(\omega - \omega_f)$$

$$-K\int\left(\boldsymbol{h} + \int\tilde{\omega}\boldsymbol{h}\,\mathrm{d}\tau\right)\mathrm{d}t - \int\tilde{\omega}\boldsymbol{h}\,\mathrm{d}t \tag{29}$$

where

$$G_1 = [D + (k + 1)KI]/k$$

$$G_2 = [(k + 1)/k]I$$

The next step is to design the momentum manager. This is accomplished easily by using a controller that satisfies the following equation:

$$\dot{x} = -K_2 x, \qquad K_2 > 0 \tag{30}$$

where

$$x = h + G_1 \int (\omega - \omega_f)\,dt + G_2(\omega - \omega_f) + K \int \left(h + \int \tilde{\omega} h \, d\tau \right) dt + \int \tilde{\omega} h \, dt \tag{31}$$

The combined attitude and momentum control law can be written as

$$u = (k + 1)[-\tilde{\omega} I \omega + T - I\dot{\omega}_f] + kG_1(\omega - \omega_f) + kK\left(h + \int \tilde{\omega} h \, dt \right) + kK_2 x - \tilde{\omega} h \tag{32}$$

Simulation results obtained using the control law just derived are presented now. The initial roll, pitch, and yaw attitude deviation from LVLH are, respectively, [2 deg, 60 deg, 2 deg]. The orbital rate is assumed to be 0.06475 deg/s. The initial angular velocity vector in deg/s is assumed to be $[0.001, -0.0637, 0.001]^T$. The gains are

$$k = 3, \qquad K_1 = \text{diag}[5E-9, 5E-8, 5E-9]$$

$$D = \text{diag}[5E4, 5E3, 5E4]$$

$$K_2 = \text{diag}[2E-3, 2E-3, 2E-3]$$

With the gains selected as shown, the inequality condition on V is satisfied easily. As shown in Figs. 2, the attitude responses are excellent, and as expected the roll and yaw momenta do not go to zero. The attitude history is given in terms of the 2-3-1 Euler angles. The oscillation of the roll-yaw CMG momenta in the absence of oscillatory disturbance is undesirable and can be eliminated easily as shown next.

Control Law 1-B

Instead of using Eq. (25), if z is defined as

$$z = (k + 1)I(\omega - \omega_f) + kh \tag{33}$$

the derivative of V given by Eq. (24) can be written as

$$\dot{V} = (\omega - \omega_f + K_1 z)^T \dot{z} + k(\omega - \omega_f)\tilde{\omega} h \tag{34}$$

If the following control law is used:

$$u = D(\omega - \omega_f) + Kz + (k + 1)\{T - \tilde{\omega} I \omega - I\dot{\omega}_f - \tilde{\omega} h\} \tag{35}$$

where D and K are as defined earlier, then

$$\dot{V} = -(\omega - \omega_f)^T D(\omega - \omega_f) + k(\omega - \omega_f)^T \tilde{\omega}_f h \tag{36}$$

Assuming that, in Eq. (36), the first term dominates the second, $\dot{V}$ is locally negative definite. Previous results can then be used to show that $(\omega - \omega_f) = \mathbf{0}$ and $z = \mathbf{0}$. Then it can be concluded that $h = \mathbf{0}$. This procedure eliminates the roll-yaw CMG momentum oscillations observed in the simulation results using control law 1-A. The design of the combined attitude controller and momentum manager parallels that described earlier. Equation (31) is modified as

$$x = h + G_1 \int (\omega - \omega_f)\,dt + G_2(\omega - \omega_f) + K \int h \, dt + \int \tilde{\omega} h \, dt \tag{37}$$

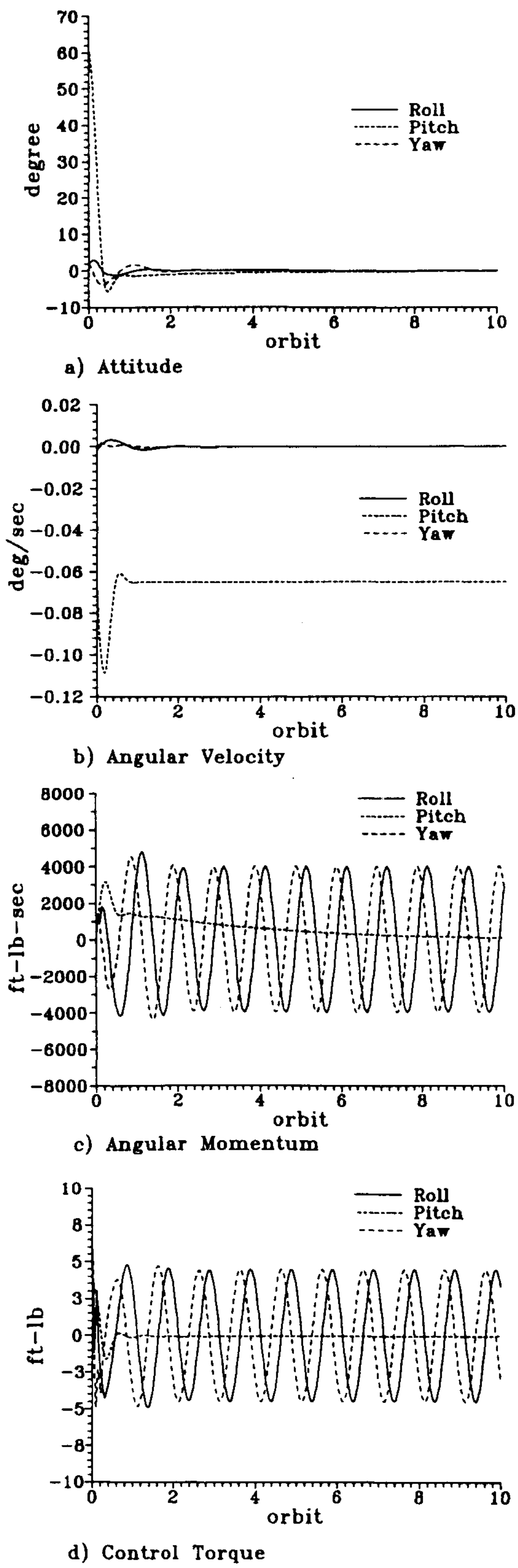

Fig. 2 Space Station response with control law 1-A, no disturbance.

and the combined attitude and momentum control law can be written as

$$u = (k+1)[-\tilde{\omega} I \omega + T - I\dot{\omega}_f] + kG_1(\omega - \omega_f) + kKh + kK_2 x - \tilde{\omega} h \tag{38}$$

Simulation results of the modified controller, using the same gains and initial conditions as before, are shown in Figs. 3. It can be seen that the responses are excellent and the momenta do go to zero.

Control Law 2-A

Consider another modification of the Lyapunov function of Eq. (13):

$$V = -k\Phi + \frac{1}{2}(\omega - \omega_f)^T I(\omega - \omega_f) + \frac{1}{2} h^T g[k + g(k+1)I]h + (\omega - \omega_f)^T g(k+1)Ih \tag{39}$$

where g is a positive scalar, and the condition $g < 1/[I_3(k+1)]$ ensures local positive definiteness of V. The derivative of this function is

$$\dot{V} = [(\omega - \omega_f) + gh]^T[(k+1)I(\dot{\omega} - \dot{\omega}_f) + \{k + g(k+1)I\}u] + k(\omega - \omega_f)^T \tilde{\omega} h \tag{40}$$

A trivial expression, $g\,k\,h^T\,\tilde{\omega} h$, is added to this equation in order to factorize it. This results in the following:

$$\dot{V} = [(\omega - \omega_f) + gh]^T[(k+1)I(\dot{\omega} - \dot{\omega}_f) + \{k + g(k+1)I\}u + k\tilde{\omega} h] \tag{41}$$

The control law can be selected as

$$u = [g(k+1)I - 1]^{-1}[-(k+1)(-\tilde{\omega} I\omega + T - I\dot{\omega}_f) - K_1(\omega - \omega_f) + \tilde{\omega} h - K_1 gh] \tag{42}$$

where K_1 is a positive definite matrix. It can be shown that, if $\dot{V} = 0$, then

$$u + \tilde{\omega} h = 0$$

$$\omega = \text{constant}$$

$$T - \tilde{\omega} I \omega = 0$$

$$T = \text{constant}$$

Thus, ω must be equal to ω_f, and $h = 0$. Hence, Eq. (7) is satisfied, and LVLH orientation is achieved. As discussed earlier, there exists four stable equilibrium points, and $V(t_0) < 3/2\, kn^2(I_3 - I_1)$ is a sufficient condition for the desired orientation.

The procedure for designing the combined attitude controller and momentum manager is as shown earlier. A new state variable is defined as follows:

$$x = (k+1)I(\omega - \omega_f) + [k + g(k+1)I]h + k\int \tilde{\omega} h \, dt + K_1 \int [(\omega - \omega_f) + gh] \, dt \tag{43}$$

where K_1 is a positive definite matrix. The momentum manager is designed as before by using a controller that satisfies

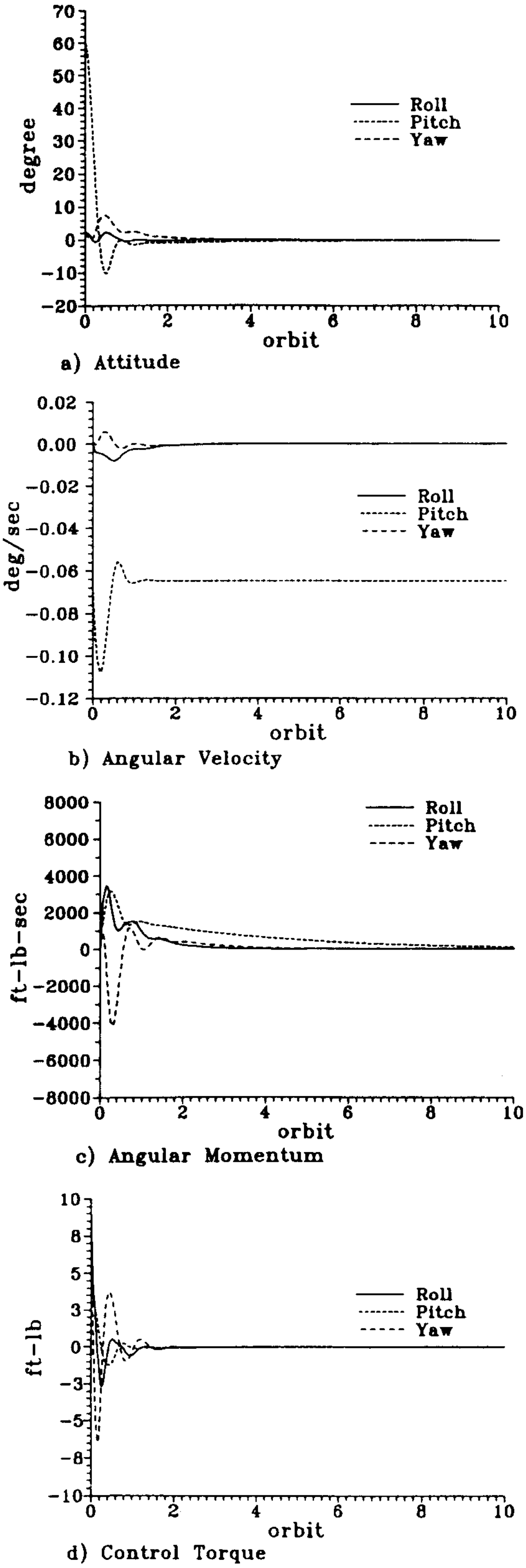

Fig. 3 Space Station response with control law 1-B, no disturbance.

the the following equation:

$$\dot{x} = -K_2 x, \qquad K_2 > 0 \tag{44}$$

The final result is a control law given by

$$u = [g(k+1)I - 1]^{-1}[-(k+1)(\tilde{\omega} I\omega + T - I\dot{\omega}_f) - K_1(\omega - \omega_f) + \tilde{\omega} h - K_1 gh - K_2 x] \tag{45}$$

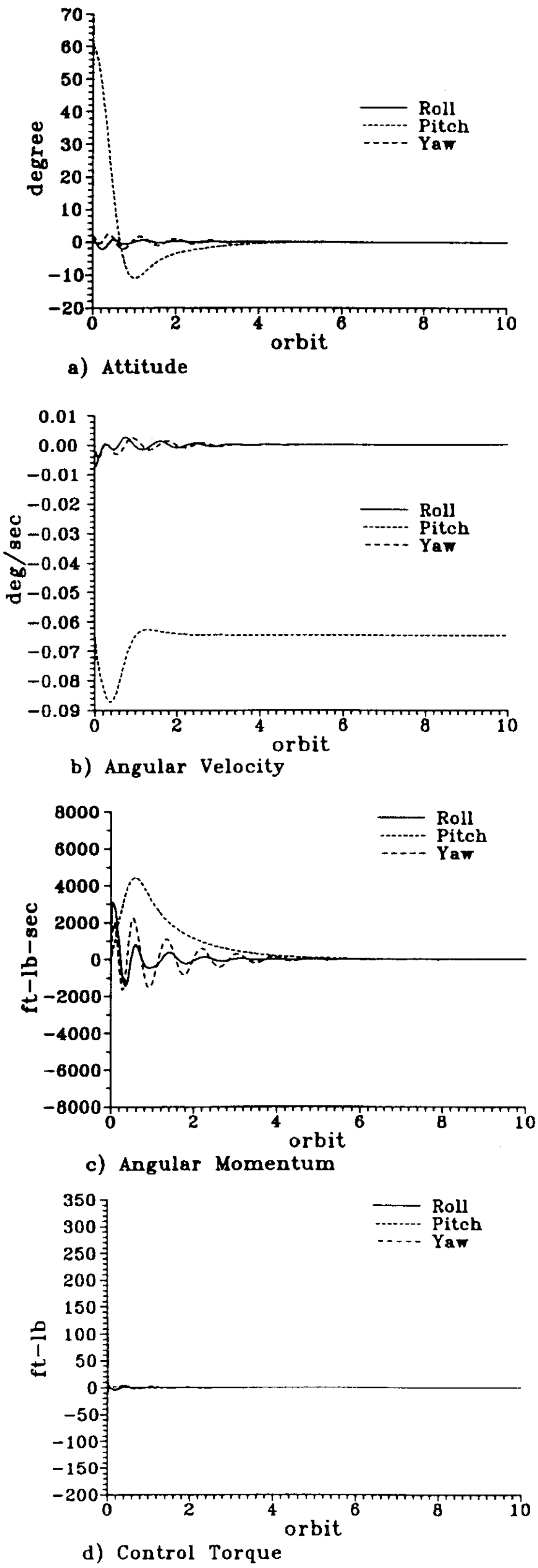

Fig. 4 Space Station response with control law 2-A, no disturbance.

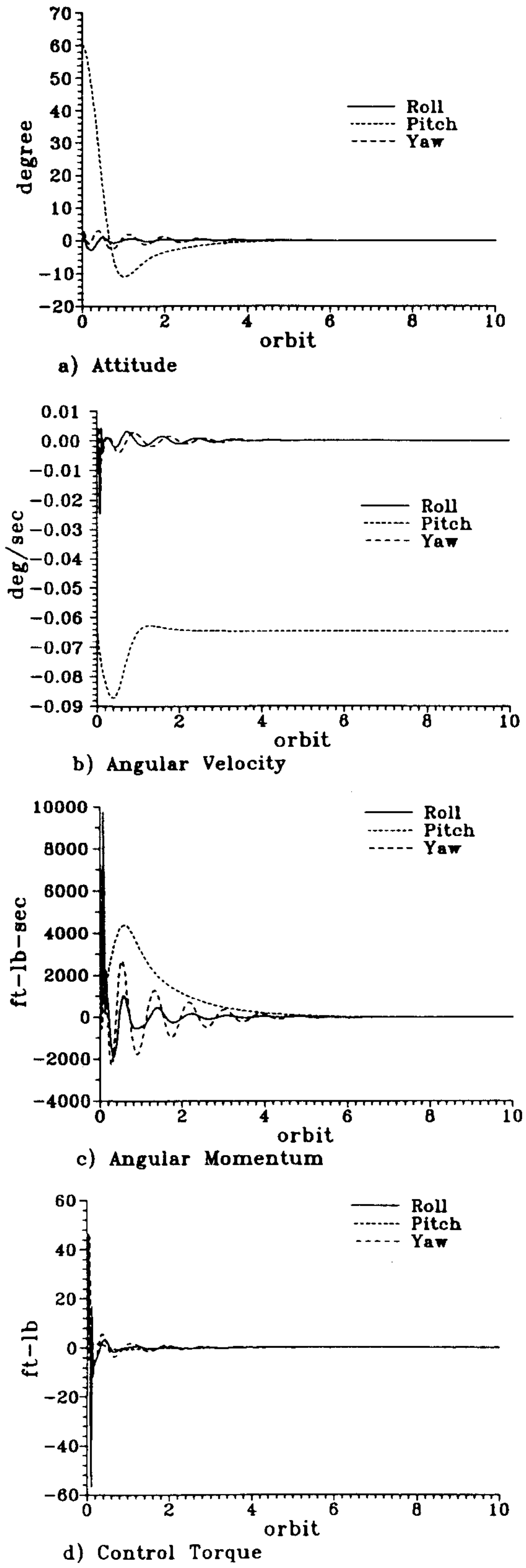

Fig. 5 Space Station response with control law 2-B, no disturbance.

Simulations of the control law were performed using the following gains:

$$k = 0.6, \qquad g = 3E - 8$$

$$K_1 = \mathrm{diag}[4E5,\ 3E3,\ 4E5]$$

$$K_2 = \mathrm{diag}[2E - 3,\ 2E - 3,\ 2E - 3]$$

The initial conditions are the same as before. The gains and the initial conditions selected do satisfy the inequality on $V(t_0)$ derived earlier. It is observed from the results shown in Figs. 4 that, although the responses are excellent, the initial roll torque is of the order of 100 ft-lb.

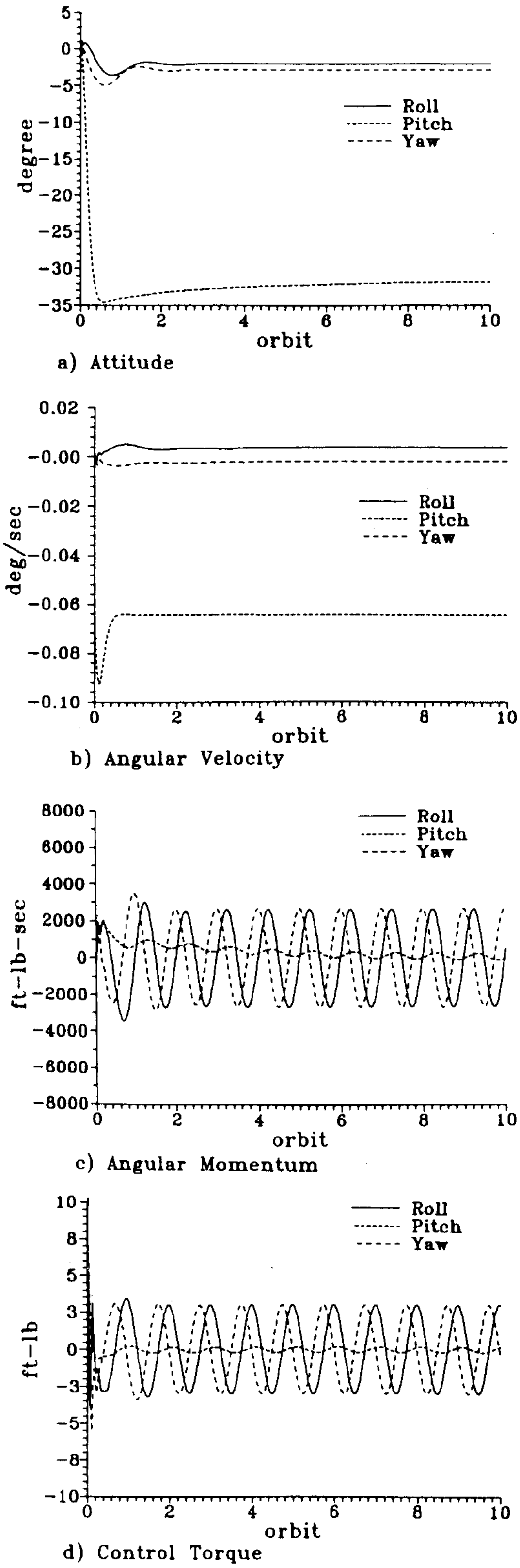

Fig. 6 Space Station response with control law 1-A, W = [1.0,1.3,1.0] ft-lb.

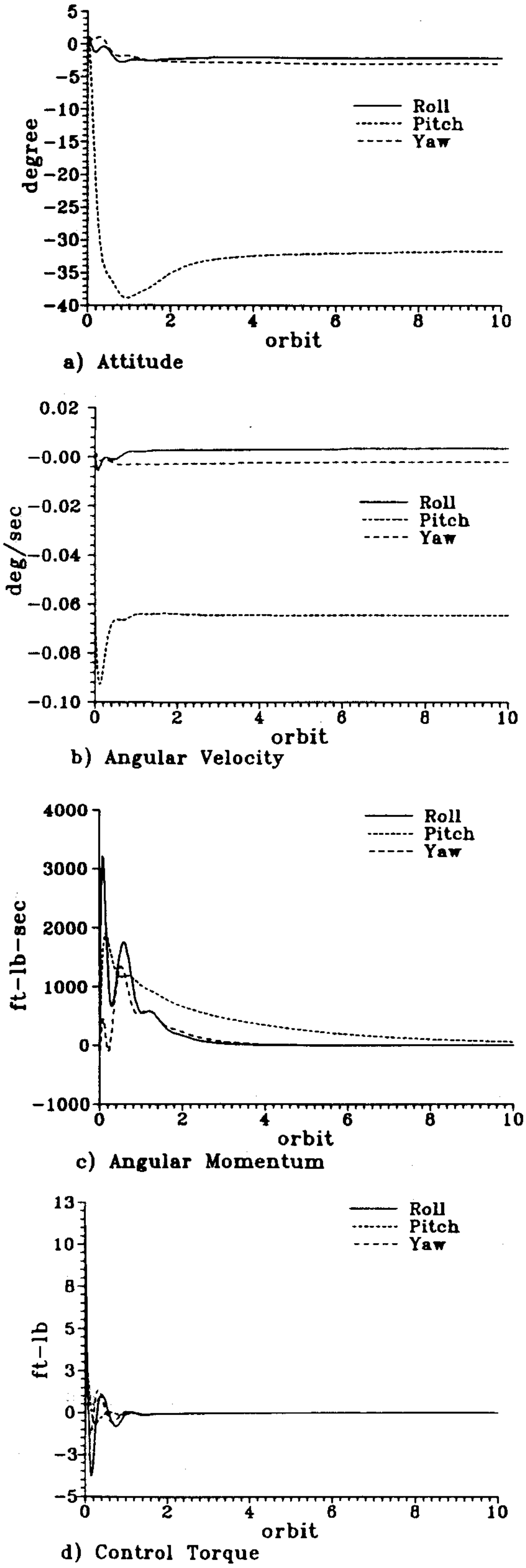

Fig. 7 Space Station response with control law 1-B, W = [1.0,1.3,1.0] ft-lb.

Control Law 2-B

To suppress the high torque demanded by the control law, a torque smoothing spline,[7] with zero magnitude and slope at the initial time and a rise time of 3000 s, is multiplied to the control torque demanded by Eq. (45). Figures 5 show the responses and control torque required. It is clear that the spline helps to attenuate the peak control torque magnitude, however, this introduces some initial transients in the responses.

Attitude Control and Momentum Management of an Unstable Configuration (with Constant Disturbance)

In this section, the effect of constant disturbances on the feedback system obtained by using the control laws derived earlier is examined. The maximum constant disturbance that can be accommodated can be obtained by solving Eq. (10). For example, the absolute value of the maximum pitch axis gravity gradient torque that can be produced to counteract

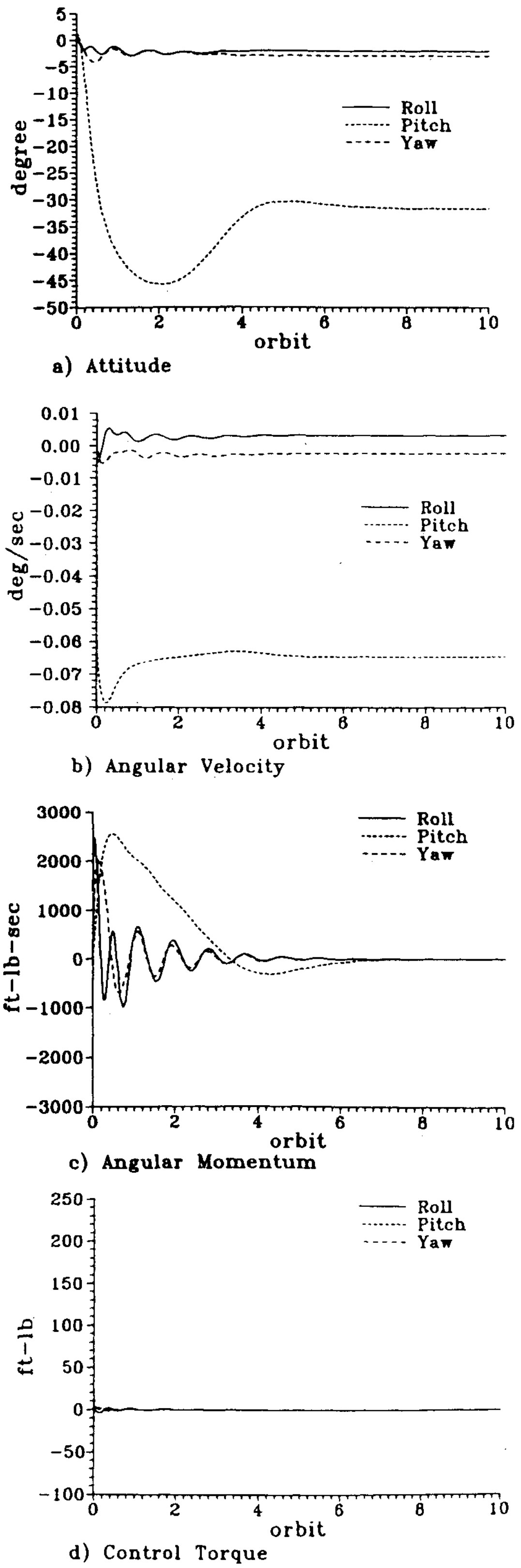

Fig. 8 Space Station response with control law 2-A, $W = [1.0, 1.3, 1.0]$ ft-lb.

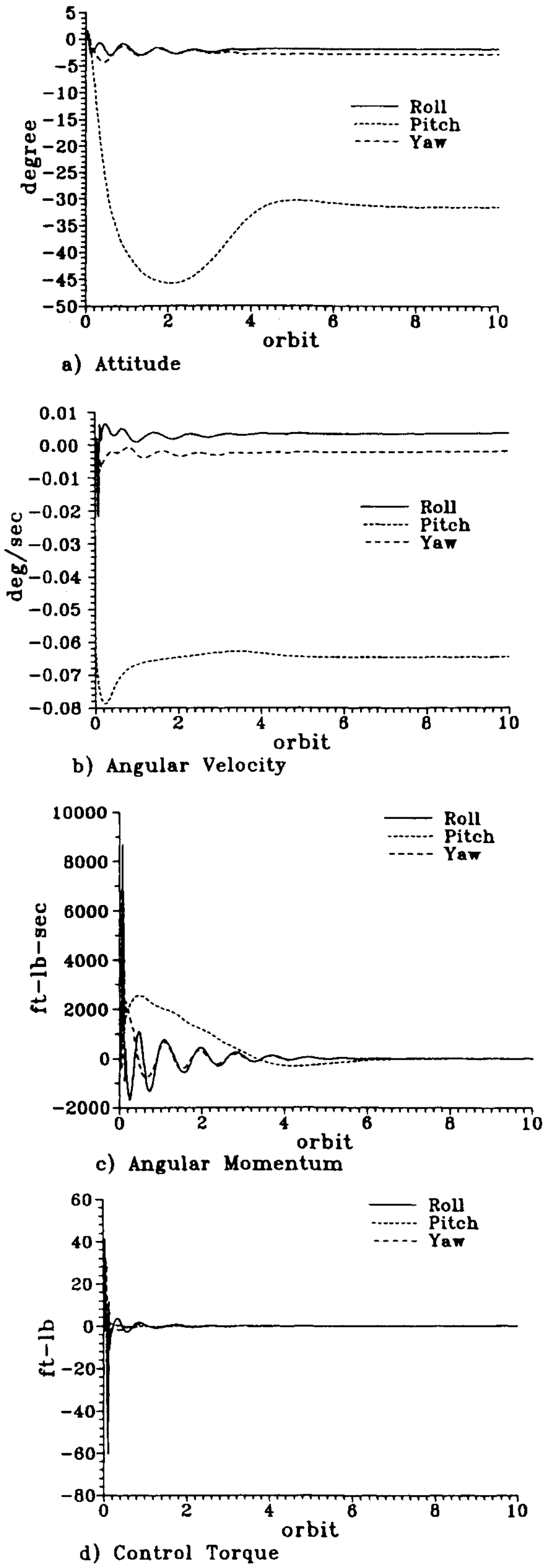

Fig. 9 Space Station response with control law 2-B, $W = [1.0, 1.3, 1.0]$ ft-lb.

external disturbance is roughly equal to 1.4 ft-lb. The disturbance value used in the simulations is $w = [1, 1.3, 1]^T$ ft-lb. The initial conditions are the same as before. The gains used for the respective control laws are also the same as for the zero disturbance case. Figures 6–9 show the responses due to control laws 1-A, 1-B, 2-A, 2-B, respectively. It is clear from these figures that all of the control laws are able to achieve the desired TEA even though the control laws were designed under the zero disturbance assumption. It is also observed that control law 1-B provides the best transient response. The torque smoothing spline used by control law 2-B results in an increase in the peak angular momentum coupled with a decrease in the peak torque magnitude.

Conclusions

Various forms of attitude and momentum control laws have been derived using the fully coupled dynamic equations of motion. The control laws have been designed using Lyapunov's second method for stability analysis and are based on system decomposition into two connected subsystems: the attitude controller and the momentum manager. The attitude controller is designed using angular momentum as the control variable. The momentum manager is designed to produce the desired momentum required by the attitude controller. First, simulation results are presented for the current Space Station configuration in the absence of disturbances. It is shown that the Space Station can be stabilized in the LVLH orientation and the momentum of the CMGs approaches zero asymptotically. The closed-loop system has four equilibrium points due to the nature of the gravitational dynamic potential. The stability boundary of the desired equilibrium point has been estimated in terms of a sufficient condition on the initial value of the Lyapunov function. The control laws designed are used in the presence of constant disturbances. It is shown that the Space Station can be stabilized about a TEA and the CMG momentum is bounded and asymptotically approaches zero. Another important conclusion is that, even under the zero disturbance assumption, if the Space Station body axes (not the principal axes) are to be aligned with the LVLH frame, RCS inputs are necessary to prevent pitch momentum buildup.

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